Para esclarecer o que a formação do professor de Matemática poderia significar para o professor de Educação Matemática, eu elaboro os seguintes temas: 1) A posição crítica da Educação Matemática que auxilia a esclarecer a própria noção de Educação Matemática Crítica; 2) Uma crença na racionalidade matemática presente na ciência e na tecnologia, que inclui a afirmação de que o progresso científico é o verdadeiro motor do progresso em todos os aspectos da vida, e que a racionalidade matemática constitui parte integral desse progresso; 3) Matemática em Ação que nos leva à compreensão da complexidade de contextos no âmbito em que a matemática pode estar operando; 4) Globalização e guetorização que posicionam a Matemática em Ação como parte de uma rede tecnológica e indicam o papel da Educação Matemática no processo da inclusão e exclusão social; 5) Solos de investigação e horizontes futuros dos estudantes, conceitos que são importante para a formação crítica do professor e que envolvem escolher entre diferentes meios de aprendizagem em cooperação com os estudantes e ser sensível aos seus motivos para aprender. Em conclusão eu argumento: 6) Crítica como um conceito aberto. Para mim, formação se refere às possíveis funções da Educação Matemática, em particular à relacionada ao processo de inclusão e de exclusão social. Entretanto, esses assuntos não são definidos por meio de qualquer quadro teórico bem elaborado. Ao invés disso, eles surgem de incertezas.

Palavras-chave - formação crítica de professores de matemática; educação matemática crítica; matemática em ação; globalização; guetorização; solo de investigação; horizontes futuros de estudantes.

---

29 Professor of Department of Education, Learning and Philosophy - Aalborg University Departamento de Educação, Aprendizagem e Filosofia da Universidade de Aalborg, Dinamarca Fiberstræde 10DK-9220 Aalborg East. Professor visitante do Programa de Pós-Graduação em Educação Matemática, UNESP - Universidade Estadual Paulista, Campus de Rio Claro. E-mail osk@learning.aak.dk
Abstract

In order to clarify what critical professionalism could mean with respect to mathematics teacher education, I discuss the following issues: (1) The critical position of mathematics education which serves to clarify the very notion of critical mathematics education; (2) A thrust in mathematical rationality in science and technology, which includes the assumption that scientific progress is the true motor of progress in all aspects of life, and that mathematical rationality constitutes an integral part of this progress; (3) Mathematics in action, which brings us to realise the complix of contexts within which mathematics might be operating; (4) Globalisation and ghettoisation, which position mathematics in actions as part of technological networking and point to the role of mathematics education in processes of social inclusion and exclusion; and (5) Landscapes of investigation and students’ foregrounds, concepts that are important for critical professionalism, which involves choosing among different learning milieu in cooperation with the students and being sensitive to the students’ motives for learning. In conclusion, I discuss (6) critique as an open concept. For me, critical professionalism refers to concerns about the possible functions of mathematics education, in particular in processes of social inclusion and exclusion. These concerns, however, are not defined through any well-elaborated theoretical framework. Rather, they emerge from uncertainties.

Key words - Critical professionalism, critical mathematics education, mathematics teacher education, mathematics in action, globalisation, ghettoising, landscapes of investigation, students’ foregrounds.
Introduction

In order to understand the possible meaning of ‘mathematics education is critical’, let me say a few words about ‘mathematics’, ‘mathematics education’ and ‘critical’.

‘Mathematics’ can refer to a school subject and also to a field of research, and the two things can be very different. While mathematics as a field of research includes a vast domain of notions and theories in development, mathematics as a school subject normally refers to a well-defined body of knowledge divided up into bits and pieces to be taught and learned according to pre-formed criteria. ‘Mathematics’ could, however, also refer to domains of knowledge and understanding, and not to any of its institutionalised forms in research or in school. Thus, we find mathematics operating in many work practices. It constitutes part of technology and design. It composes part of procedures for decision making. It is presented in tables, diagrams, graphs; thus we can experience much mathematics leafing through the daily newspaper.

‘Mathematics education’ can refer to activities taking place at all school levels, from kindergarten to the university. It can refer to teaching and learning processes in engineering, in economics, in a wide range of technical disciplines. Mathematical teaching and learning can take place in many out-of-school situations, often not considered to have anything to do with mathematics. We can think of banking, accounting, computing, design, not to mention shopping and household management.

And ‘critical’, what could that mean? Here we may remind ourselves about a common use of ‘critical’ in medicine. The situation of a hospitalized patient may be critical. This means that his or her situation could go ‘either way’; and it makes a big difference which way it goes. Mathematics education – understood in a broad sense, but also as a school subject – can be acted out in very different ways. In this sense, I talk about mathematics education as being critical.

The critical position of mathematics education is a challenge for all teacher education. Being aware of and facing such challenges, one way or another, is a characteristic of what I call critical professionalism. In order to clarify what this professionalism could mean with respect to mathematics teacher education, I discuss the following issues: (1) The critical position of mathematics education. This I have already referred to, but I will expand on this idea as it serves to clarify the very notion of critical mathematics education. (2) Mathematical rationality in science and technology. A thrust in this rationality includes the assumption that scientific progress is the true motor of progress in all aspects of life, and that
mathematical rationality composes an integral part of this. A blind trust in mathematical rationality must be questioned by critical professionalism. (3) *Mathematics in action.* A discussion of mathematics in action brings us to realise the complex of contexts within which mathematics might be operating. (4) *Globalisation and ghettoisation.* It is generally agreed that, in any teacher education, it is important to consider the context of teaching and learning. By pointing out the close connection between globalisation and ghettoisation, I try to indicate the scope of contextual issues that critical professionalism could address. (5) *Landscapes of learning and students’ foregrounds.* Any teaching and learning process can be organised in a variety of ways. Which may be considered the most appropriate depends on many issues. What is important for critical professionalism is the readiness to choose a path among different learning milieus in cooperation with the students, and being sensitive to the students’ foregrounds. (6) In conclusion I will discuss: *Critique as an open concept.*

Naturally, there are more issues to be addressed in order to clarify what critical professionalism with respect to mathematics teaching could mean. In particular, there is much to be said about mathematics that is relevant for the clarification of any professionalism, critical or not. Furthermore, there is a wide range of aspects concerning the professional organisation of teachers, which is often addressed as part of professionalism. Here, however, I concentrate on issues through which I try to characterise critical professionalism. The discussion will to a large extent be based on my book *Educação Crítica: Incerteza, Matemática, Responsabilidade* (Skovsmose, 2007), which includes a broader presentation of concerns regarding critical education.

**The critical position of mathematics education**

In literature and in films, we find horrible examples of mathematics education, often personified by a mathematics teachers who dominates the students and punishes, through an icy irony if not physical punishment, those who do not grasp even the simplest mathematical concept, leaving the student humiliated.

Examples like this provide a bleak picture of mathematics education, but let us take a more careful look at what can be called the *school mathematics tradition.* In this tradition, mathematics exercises play a crucial role. If we consider research in mathematics education, little support can be found for the claim that this is a particularly productive approach. Mathematical creativity is not cultivated through this tradition. In fact, it might appear that some deep socioeconomic irrationality is maintained as part of mathematics education. But how could it be
that educational systems worldwide cling to this apparent dysfunction? Or could it be that we may instead be dealing with a kind of learning-for-obedience, which might fulfils the needs of the dominant economic order of the labour market?

Let us take a more careful look at a possible exercise: “A shop is offering apples for either 0.12 Euro a piece or 2.8 Euros for 3 kilos. There are 11 apples per kilo. Calculate how much Peter will save if he buys 15 kilos of apples instead of buying them individually.” As with most other exercises from the school mathematics tradition, this exercise was just formulated at a desk, without the need to do any empirical investigation. Furthermore, the information given can be considered to be exact. When doing the calculation, one can be sure that there are 11 apples, and exactly 11 apples, per kilo, just as we can be sure that the price is exactly 0.12 Euro for one apple. That we are dealing with two different kinds of truth is of no significance, and need not be addressed in any way in calculating the solution. Any information provided in the text of an exercise can be considered exact and absolute.

The information provided in the exercise is sufficient and necessary. Based on the given information, it is possible to calculate the one and only correct answer. It is unnecessary to seek additional information. Certainly there is no need for students to leave the classroom in order to search for supplementary information. One can also be sure that all information provided must be used, at least the information presented in numbers. In this sense, an exercise establishes a micro-world where all measures are exact, and where the information given is necessary and sufficient in order to calculate the one and only correct answer.

In the process of solving the long series of exercises that has been assigned to them, students may be learning something which does not necessarily have to do with a deeper mathematical understanding. Exercises could be formulated as: “Reduce the expression…!” “Solve the equation …!” “Find x, when …!” “Calculate the area of …!” They often appear take the form of a long sequence of orders. Could it be that the school mathematics tradition serves as an introduction to work processes where the ability to carefully carry out prescribed behaviours of any kind is important? Could it be that such a prescription-readiness is functional for innumerable job-functions in our society, and that the school mathematics tradition serves perfectly well to prepare future workers for this prescription-readiness? Would such readiness not be functional when working as an assistant in a bank, shop, etc? Furthermore, could it be that the school mathematics tradition forces many students to assume they are unable to learn mathematics, and as a consequence, that they cannot have much hope of acquiring competencies that are valued in the labour market? And that they therefore must be prepared to
accept low paid jobs? Could the school mathematics tradition represent a way of thinking that facilitates submission to power?

Through such considerations, we could come to associate mathematics education with learning-for-obedience. But what to think, then, of mathematics education and ‘empowerment’? This expression can be interpreted in different ways with reference to mathematics. A classic notion of intellectual empowerment assumes that mathematics education cultivates the best part of human thinking. Such a conception draws from a long tradition in philosophy and epistemology. Since Plato outlined his epistemology, knowledge and certainty have been associated; and the most splendid example of how to obtain certainty was demonstrated through mathematics. Here mathematics establishes an insight into eternal structures, exemplifying what sublime intellectual empowerment could mean. A pragmatic interpretation of empowerment points out how mathematics education can provide people with qualifications that make it possible for them to function as citizens and secure an interesting job. Empowering a person means to prepare him or her for participation in and enjoyment of the benefits of society. For a majority of people, this includes the ability to operate in a functional way in the labour market. Mathematics education, it is claimed, guarantees a good position in the labour market for many people. Empowerment can also be thought of in a radical and political way. There are many examples of educational practices that involve the development of teaching and learning around projects where mathematical insight combines with critical investigations of socio-political issues. This brings forth the idea that empowerment through mathematics education can mean reading the world as being open for changes.

In other words, some observations lead us to see mathematics education as preparing students for a prescription-readiness, while others make us see mathematics education as empowering. Mathematics education might mean disempowerment; it might mean empowerment. Both statements, contradictory as they might appear, can find support in a rich array of justifications. This brings me to claim that mathematics education is critical. This education can be acted out in radically different ways. There is no ‘essence’ in mathematics education which ensures that it will serve particular attractive functions (although this may be the case), nor any particular problematic function (although this may also be the case). This education could simply go ‘either way’.

To be concerned about the critical position of mathematics education is, for me, a characteristic of critical mathematics education. This critical position is also a challenge for teacher education. Education of mathematics teachers cannot be based on a simple assumption that teaching mathematics is justified
through its intrinsic goodness. Concern regarding this multitude of possible functions of mathematics education is an important aspect of critical professionalism among teachers. It becomes important in all mathematics teacher education to address the critical nature of mathematics education: its potential for empowerment as well as for disempowerment, keeping in mind that the notion of empowerment is impossible to delineate, as is the notion of disempowerment.

**Mathematical rationality in science and technology**

The so-called Scientific Revolution launched the idea that science leads to progress. Let me just recapitulate some elements of this revolution. Inspired by ancient Greek philosophers, in particular the Pythagoreans, Nicolaus Copernicus (1473-1543) presented a heliocentric world picture. This was a radical alternative to the geocentric description provided by Ptolemy, which had been authorised by the church as the proper picture of the universe.

Copernicus assumed that the movements of the planets were circles, but through a careful investigation of the movement of the planet Mars, Johannes Kepler (1571-1630) suggested that the planets were moving in ellipses, with the sun as one of its loci. Through Kepler’s observation, mathematics obtained a particular epistemic position. It was possible to describe exactly the orbits of the planets through mathematics. In this way, it was assumed that mathematics represented the structures of nature. The Pythagorean idea that everything is numbers received a powerful re-interpretation. Mathematics could be seen as providing the master plan for God’s creation of the world; and it should be remembered that an unquestioned belief in God’s existence dominated the outlook of all the people who contributed to the Scientific Revolution. Only later did atheism emerged as an intellectual possibility.

Galileo Galilei (1564-1642) also found that mathematics played a particular role in the study of nature, being essential for formulating the laws of nature. He distinguished between appearance and reality, or in other words, between primary and secondary sense data. While the primary sense data refer to position, movement, shape, and weight (as well as to the number of entities), the secondary data refer to colour, smell, sound, taste and texture. According to Galilei, only the primary qualities were significant for understanding nature, and it is precisely these qualities can be depicted mathematically. While the secondary sense data signifies how our senses organise our experiences, then mathematics helps to delineate the reality behind these experiences. In short, mathematics brings forth the essence of nature.
Isaac Newton (1642-1727) provided the elegant completion of the scientific revolution by formulating the laws that govern all kinds of movements – on earth as well as in heaven. The essential point is that we are dealing with the same laws for both Earth and heaven; the same explanation applies for the trajectory of a stone that is thrown and the movement of the Earth around the sun. Newton brought the whole picture together with the notion of gravity. Any two units of mass, no matter where they are located in the universe, are attracted to each other by a force that is proportional to the product of the two masses and inverse to the square of their distance. This law of gravity operates throughout the entire universe.

The Scientific Revolution provided mathematics with a crucial position. Mathematics ensures the basic insight into nature. It was well known that mathematics was used to provide the beauty and correct proportions of any architectural construction. Saint Peter’s Cathedral in Rome was just a recent example of detailed mathematically-formulated architectural design. And now it became evident that God, as the architect of the universe, had also used a mathematical blueprint. Mathematical insight was important for establishing insight into God’s creations. In fact, mathematics represented an overlap between human knowledge and God’s knowledge and wisdom. Although God would grasp things must faster than human beings, his insight would not contradict mathematics.

The Scientific Revolution brought forward not only the idea that science engenders progress, but also that scientific progress is the true motor of progress in all spheres of life. Since science-based development takes the form of technology, then technological development became framed in a most positive discourse: humankind was surrounded by a hostile nature, but through technology, nature has been tamed. It is impossible to imagine any technological innovation without a profound amount of mathematics being brought into operation. From being a language for understanding nature, mathematics becomes the language of technology, whatever domain we might have in mind. The thrust in progress came to embrace a thrust in mathematical rationality in all possible domains.

However, the simple relationship between technological development and social progress has been questioned. Lately, the notion of the risk society, as suggested by Ulrich Beck, emphasises that technology has enveloped humankind in a techno-nature, which includes technology-produced risks. The creation of atomic energy may serve as an illustration. An atomic power plant represents an enormous source of energy, but it also represents a new form of risk structure. A catastrophe might not occur, but it could. Even the most unlikely catastrophes might happen. Mathematical rationality is an indispensable resource for all these
forms of technological constructions, initiatives, and decision making that form the risk society. As a consequence, a mathematical rationality can also be seen as a doubtful rationality. It is a rationality which can provide very important innovations, but it can also bring about catastrophes. It is an uncertain rationality.

The idea that mathematics teachers should function as ambassadors of mathematics has dominated much mathematics teacher education. However, mathematics does not operate merely as a simple rationality of progress. Mathematics is operating in all forms of technical disciplines. It forms part of technological risk production. A principle element of any critical professionalism is to distance oneself from the content to be taught and learned in order to reflect critically on it. It must be addressed through reflections. In order to illustrate the scope of this task, I will comment on mathematics in action.

Mathematics in action

My discussion of mathematics in action is inspired by two ideas from the philosophy of language: Linguistic relativism and a performative interpretation of language. If we consider mathematic as a language – a language of science, a language of technological rationality – these two ideas can be applied to it.

Linguistic relativism, as suggested by Sapir and Whorff, introduces the idea that language might not only provide a description of what is seen, but also shape a world-view. Language provides a grammar, not only for what to say and not to say, but also of what world we will experience and not experience. Linguistic relativism can be rephrased in Kantian terms: what we experience is not things-as-such, but things-for-us. What we experience and what we are able to grasp is structured by categories, without which we would have no experiences. According to Kant, such categories have a permanent nature identifiable only through a transcendental philosophy, as conducted by Kant himself. However, according to linguistic relativism, such categories are historically and culturally developed. They are integrated in the basic grammar of language. The basic format of our life-world is, thus, an expression of manufactured linguistic categories. This brings language into a crucial position for understanding what we will refer to as our reality. Linguistic categories provide a formatting of our life-world.

A performative interpretation of language was introduced by Austin and Wittgenstein: Something is done my means of language. In Austin’s terminology, any utterance of a statement includes three aspects: content, force and effect. Statements, expressions, formulations, questions, etc. include acts. Things can be done through words.
When we combine the two ideas, that language provides a world view, and that it includes actions, the path is opened for discourse theory, as well as for an investigation of mathematics in action. I have presented the idea of mathematics in action in different ways. I do not try to provide any standardised presentation of this type of action. Here I limit myself to citing the follow four dimensions:

(1) *Technological imagination* refers to the possibility of conceptualising technological possibilities. A prominent example of mathematics-based technological imagination is the construction of the computer. It was not even possible to imagine the operation of an electronic computer without mathematics at hand. Thus, the Turing Machine provided a theoretical simulation of how a computer would operate, and it was possible to investigate all possible details of this machine. The Turing Machine was, however, only a mathematical construct, defined through its particular algorithmic operations. It was not a physical, existing machine. It was possible to analyse the capacity of the Turing Machine, and in this way to identify limits which no computer could surpass - and to do so even before the first computer was built. The whole conception of the Internet can also serve as an example of what mathematics-based imagination could mean. There is no way to imagine the functioning of the Internet by means of even an elaborated common sense; mathematics is needed. I see technological imagination as defined through mathematics in action.

(2) It is possible to establish a wide range of *strategies* and patterns of operation and decision making by means of mathematics. In economics, advanced mathematical models are brought into operation. Models are used by any government to simulate implications of economic decisions. In this way, economic policymaking become experimental. Naturally, any big company operates with simulation models as well. However, the result of any model-based experimentation need not be repeated by reality. Any model contains simplifications. Some parameters might have been ignored; some connections formulated by the equations which constitute the model may be fictitious. Nevertheless, mathematics-based experimentation dominates economic decision making at both the macro and micro levels.

(3) Mathematics in action comes to form part of *reality*. A mathematical model is not just a detached description of reality. One can think of the way prices of many items are defined – think of the price of a mobile phone, a TV set, a car, a house. In many such cases, one should not think of a price as defined by a particular number on the price tag, but as a set of conditions for a sequence of economic transactions. And the very identification of such conditions is based on mathematical modelling. In such a case, a model serves not as a detached description...
of forms of payment, but rather defines the payment. The model constitutes part of real-life economic transactions. It defines aspects of the life conditions for, say, a family who is struggling to pay for a car at fixed rates for a three-year period. In this sense, mathematics in action becomes real.

(4) Normally, actions call for reflections, as an ethical dimension seems included in the judging of any actions; however, mathematics-based actions often have the appearance of being neutral. Such actions could present themselves as the only ‘objective’ things to do. Thus, a political discussion could refer to ‘facts’ provided by an economic model. Calculations could demonstrate the need for a company to lay off workers in order to keep within the budget. Through a reference to an underlying mathematical model, or more generally to calculations, a neutralisation takes place, as mathematics-based actions easily become enveloped in a language of objectivity.

Certainly many more aspects of mathematics in action could be addressed, but the aspects presented may serve as an entry into a discussion of the mathematics-power complex. For me, it is important that critical professionalism in mathematics teacher education address how knowledge and power might be operating and interacting, and this domain of social dynamics can be addressed through particular investigations of mathematics in action. One aim is to leave behind any blind trust in the intrinsic goodness associated with mathematical rationality. Naturally, this does not imply that one should try to eliminate the use of mathematics. The point is that any form of mathematics in action, like any other form of action in general, should be the focus of critical reflections.

Globalisation and ghettoisation

‘Globalisation’ is a popular term, although far from being a popular phenomenon. Globalisation can refer to a new order of domination and exploitation; it could be an economic order supported by military supremacy, making it possible to promote multinational companies in their search for raw material, cheap labour, and new markets. Despite being multinational, such companies can be firmly settled in the world’s richest countries. Globalisation can refer to the network of production, where products are fabricated in poor regions by a cheap labour force and delivered to the affluent areas. Thus, globalisation may refer to the world-wide development of informational capitalism.

The notion of globalisation can also be guided into a more ‘human’ interpretation, leaving out the direct aspects of economic and military power. It can be elaborated in terms of a growing concern for each other based on new
forms of communication. News is spread globally; we become aware of problems all over the world. It is possible, through the Internet, to communicate observations and opinions in ways that make it difficult for governments with dogmatic, if not dictatorial, aspirations to maintain a firm grip on the population. The universal stream of information means that a variety of concerns become universal. In the same sense, people all over the world experience the same song competitions, the same sports events, the same films, not to mention the same Big Brother.

Globalisation can mean inclusion, referring to the growing concern for the welfare of the whole world. But globalisation also includes processes of exclusion. Certain groups can be marginalised, and as a consequence, I see ghettoisation as an aspect of globalisation. By ghettoisation I refer to social and economic processes that isolate groups of people from the economic transactions of the globalised, informational economy. Ghettoisation takes place in any society; thus, metropolises the world over are experiencing an explosive growth in the number of favelas. So, in my vocabulary, globalisation includes the grammar of ghettoisation.

Mathematics-in-action plays a particular role in establishing the technological underpinnings of the network society, including both globalisation and ghettoisation. Information and Communication Technologies (ICT) would not be successful without mathematics in operation. ICT is a form of materialised, formal algorithms. All kinds of strategies that define international business interactions are based on ICT networking. This includes the accelerated operations at the stock market and the organisation of financial transactions in all of its forms; it includes forms of management, and of organisation of production. Any production can be described as a network of production lines, which start where raw material is provided, and pass through knots of production units, and terminate where the product reaches the market. This production network can continuously be reorganised and redirected according to the possibilities for maximising profit. This reorganisation also means a restructuring of risks, as some areas may become dumping grounds for risky forms of production. This relocation of certain forms of production is an integral part of the informational economy. The whole process might appear gentle and clean, as the whole decision-making process can be expressed in cost-benefit analyses conducted in formal calculations. In short: mathematics-in-action is a principal element of networking technologies.

This brings us to mathematics education. At the most general level, this education can be interpreted as the universal preparation of young people to acquire certain competencies, which they may need to take advantage of further career opportunities, and that appear necessary for the proper functioning of technological and socio-economic superstructures included in a wide variety of different practices.
As mathematics can be seen as a universal language, mathematics education can be seen as a universal form of socialisation of students into certain perspectives, discourses and techniques which are imperative if we are to operate with the present technological and economic framework as a given and not as something that can be questioned. Mathematics education is basic to a wide range of technology and work practices that define processes of globalisation.

I feel it is important that critical professionalism, with respect to mathematics teacher education, include a broad perspective on the whole context of mathematics education. The actual functioning of mathematics education depends on the context in question. Keeping in mind that globalisation includes ghettoisation, it should not be surprising to find mathematics education operating in processes of inclusion and exclusion: “Mathematics … has … been cast in the role as an ‘objective’ judge, in order to decide who in the society ‘can’ and who ‘cannot’. It therefore serves as the gate keeper to participation in the decision making processes of society. To deny some access to participation in mathematics is then also to determine, a priori, who will move ahead and who will stay behind.” (Volmink, 1994: 51-52) This statement by John Volmink seems to me more current than ever. Mathematics education helps to establish a division between those who are included in and those who are excluded from the informational society. And we should remember ‘inclusion’ could mean ‘inclusion under certain conditions’. Inclusion in the labour market might require not only some testified competencies but also certain attitudes, like a prescription-readiness and learning-for-obedience. This is a principal concern for any critical professionalism in mathematics teacher education.

A teacher from Brazil has described to me what it means to be a teacher in a provincial city where the majority of her students between 11 and 15 years of age have family members in prison. They are close to a tough criminal milieu. What could the learning of mathematics mean to such children? What are their interests? She told me that they like copying very much. Whatever she says and writes on the blackboard, they like to copy into their books. However, one thing was even better than copying from the blackboard: the computer was a highly motivating factor. The students were invited to experiment with the Cabri software programme. They liked it, although the teacher doubted that it was the Cabri programme, or any mathematical programme for that sake, that had caught their interest. It might be the computer itself that had engaged the students. They could touch a keyboard.

What could empowerment mean in this context: allow the students to copy from the blackboard, or let them work with the computer? What interpretation of
empowerment is relevant in this case? I feel that we should be careful to avoid making specific assumptions about what serves as empowerment in specific situations. Instead, it is important to explore educational possibilities. In order to do so, a variety of ‘milieus of learning’ might be considered.

One can imagine very different milieus of learning established in the mathematics classroom. Some, like those defining the school mathematics tradition, are dominated by prefabricated exercises that the students have to solve, and that have one, and only one, correct answer. Some milieus are dominated by the teacher’s presentation at the blackboard and students’ copying what is written there. Other milieus can be organised around investigations of mathematical issues. Or they may refer to mathematics in real-life situations. One can organise activities around geometrics software. One can establish classroom milieus around group work or project work. One can establish milieus with different forms of communication among students, and between teacher and students.

I do not believe any particular learning milieu to be the most appropriate for organising classroom practices. I normally suggest that teacher and students travel between different milieus, although often I have suggested that different forms of landscapes of investigation might provide milieus which could bring new dynamics to the learning of mathematics and to the interaction in the classroom. I find that critical professionalism should include a capacity of the teacher to operate in different milieus of learning, and to move between such milieus, and to do so in collaboration with students.

It is important for the learners to be involved in the process of learning. Naturally, one can learn many things, even when one is forced to do so. However, I find that a certain quality of learning emerges when students’ intention to learn becomes a driving force in the process of learning. One of the ideas behind introducing landscapes of investigation is to make it possible for students to identify a variety of motives for learning.

This brings me to the subject of being attentive to the students’ foregrounds. In referring to a person’s foreground, I mean the opportunities that the socio-political and the cultural situation make available for the person; not the opportunities as they might exist in any objective form, but as they can be experienced by the person. And certainly such opportunities need not be perceived in any unified or consistent way. A foreground might include different and also contradicting perspectives. Motives for learning relate to the foreground of the students. So in order to provide motives for learning, it is important that the content,
one way or another, be related to the foregrounds of the students. Thus, meaningfulness of classroom activities, from the perspective of students, concerns to what extent they might experience relationships between what is taking place in the classroom and situations outside the classroom, including situations they perceive as possibly belonging to their future. Landscapes of investigation may be organised in such a way that they relate to students’ foregrounds.

Foregrounds are experienced but far from freely constructed. A foreground might be ruined. Ghettoisation, which is an integral part of globalisation, is one of the main causes for ruining the foreground of some groups of students. Think of the foreground experienced by students from a poor neighbourhood, which might not leave open many possibilities for escape; or of the students from that provincial town in Brazil where the majority have family members in jail. Like anybody else, such students might also search for motives for learning by trying to relate the content presented to them in the classroom to the content of further work situations they might be looking forward to. We should not assume any ready-made answer to what this could mean. Touching a keyboard could have a particular significance. Nevertheless, their search for meaning in classroom activities might be in vain. Maybe they are only able to relate the classroom practices to situations they do not even dare to imagine might belong in their foreground: studying at a university, becoming an engineer, working in a bank. Ghettoisation could rob them of possibilities to establish motives for learning.

Critical professionalism includes sensitivity to the students’ foregrounds. This does not mean that such professionalism could change the realities of students. This would be a demand that reaches far beyond any teacher professionalism, critical or otherwise. But critical professionalism must demonstrate an effort to relate activities in the classroom with issues that may come to occupy the students, and which could establish the students’ intentions for learning as part of the classroom practice.

I have been involved in different studies of students’ foregrounds, for instance students from a favela (slum) and indigenous students in Brazil. To indigenous people in Brazil, mathematics education might signify opportunities for moving across cultural boundaries. As pointed out in an interview with indigenous students, mathematics is significant for studying medicine, and health problems are a crucial issue in the indigenous environment. The relevance of mathematics for farm work was also emphasised; it was seen as relevant for dividing up the harvest and making business deals. In other words, different foregrounds give very different meanings to the learning activities. Critical professionalism pays a special attention

to the relationship between the content in mathematics education and the foregrounds of the students.

This professionalism does not presuppose any particular affluent context for learning. It can try to establish landscapes of investigation in any possible context.

**Conclusion: Critique as an open concept**

Previous formulations of critical education have found inspiration in Critical Theory. However, I find it is important that the discussion of critical education be established in a new format.

In *Knowledge and Human Interest* from 1968, Habermas pointed out different knowledge-guiding and knowledge-constituting interests. Natural sciences incorporate a technical interest organising knowledge from a perspective of goal-oriented changes. The knowledge-constituting interest of the humanities aims at bringing about understanding, while an emancipative interest constitutes knowledge within the area of social science. However, the scientific reality of social science was dominated by a positivist conception, implying that the social sciences were dominated by the technical interest of the natural sciences; and as a consequence, the social sciences came to serve the well-established political and economic order.

This discussion of knowledge-guiding interests had an impact on the conception of education. Even though education could be thought of as a humanistic discipline, the inspiration from Habermas introduced emancipation as a defining element of critical education. Naturally, it also brought substantial difficulties regarding the interpretation of what emancipation might mean within the domain of mathematics and science education. According to Habermas’ analysis, natural science was guided by a technical interest, of which domination is an integral part. As mathematics represents the logic of domination, what sense can be made, then, if any, of critical mathematics education? It could almost appear to be a conceptual contradiction.

Critical Theory established critical thinking outside some of the principal assumptions operating within Marxism. However, Critical Theory may nevertheless have incorporated a variety of assumptions that were passed on into critical education. To illustrate what I mean, we can consider the notion of emancipation. In the original format of critical education, the notion of emancipation occupied a prominent position. Critical education should be guided by an overall interest in emancipation, and certainly not by a technical interest; nor would an interest in understanding – guiding education as a humanistic discipline – be sufficient. But
what does it mean for educational practice to be guided by an emancipative interest?
I find that critical education has suffered from too many attempts to answer these questions in the form of simple guidelines for organising an educational practice: teaching should be organised in projects that relate directly to the every-life of the students, etc. Naturally, such a proposal might be useful in some situations. However, I do not think it possible to extract guidelines from any Critical Theory regarding how to operate with mathematics, or any topics for that matter, in the classroom. In fact, I do not believe there is anything to be called a ‘critical theory’, if we take ‘theory’ in any regular sense of the word.

Instead, I believe that critical mathematics education can emerge from an uncertainty with respect to how mathematics education might operate in different contexts. For me, critical professionalism refers to concerns about the possible functions of mathematics education. It reflects concerns about the possible functions of mathematics in action, of how mathematics may operate in different technological or every-day practices. It represents concerns about overall socio-political and economic issues as expressed in discourses about globalisation and ghettoisation. It represents concerns about how to organise particular milieus of learning, considering the foregrounds of the students. The concerns are not defined through any well-elaborated theoretical framework, and there is no straightforward notion of empowerment that could provide educational guidelines. I believe that the meaning of the notion of ‘critique’ in ‘critical education’ and in ‘critical mathematics education’ might be found in concerns emerging from uncertainties.

Acknowledgements

I want to thank Miriam Godoy Penteado for critical comments and suggestions for improving preliminary versions of this paper, and Anne Kepple for completing a careful language revision.

References

